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Modification on Adomian Decomposition Method for Solving Fractional Riccati Differential Equation

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Abstract: In this paper we introduced a new approach to solve fractional order Riccati differential equation that called Adomian Decomposition Method- Restrictive Padé (ADM-RP) which is anew fast and efficient method that approximate the series solution given by ADM to a fraction function called Restrictive Padé approximation using mathematica package the solution by ADM-RP gives better solution.

Keywords: ADM- Restrictive Padé- Fractional Differential Equation- Riccati Differential Equation

I. INTRODUCTION

The following non-linear fractional order Riccati differential equation

$$D_*^{\alpha}\, y(t) = A(t) + B(t)y + C(t)y^2 \quad t > 0, \qquad n-1 \le \alpha \le n \tag{1}$$

Subject to the initial conditions

$$y^{(\mathrm{k})}(0) = c_k \quad k = 0,1,\dots,n-1 \tag{2}$$

where α is the fractional order derivative and n is an integer. $A(t), B(t), C(t)$ are given real functions c_k $k = 0,1,\dots,n-1$ is constant. Riccati differential equation is a class of nonlinear differential equation of much importance and plays a significant role in many fields of engineering and applied science.

An advantage of the decomposition method is that it can provide analytical approximation to a rather wide class of linear nonlinear equations without linearization, perturbation, closure approximations, or discretization methods which can result in massive numerical computation[1].

The Adomian decomposition method was introduced and developed by George Adomian in [2–3]. The Adomian decomposition method (ADM) is a semi-analytical method for solving both ordinary and partial nonlinear differential equations. Also ADM used for solving the fractional order differential equations. The crucial aspect of the method is employment of the "Adomian polynomials" which allow for solution convergence of the nonlinear portion of the equation, without simply linearizing the system. These polynomials mathematically generalize to a Maclaurin series about an arbitrary external parameter; which gives the solution method more flexibility than direct Taylor series expansion.

Massive equations have been solved by ADM [4-7]. the application of the ADM in solving nonlinear wave-like equations with variable coefficients was present in [4].(ADM) for solving nonlinear integro- differential equations [5].a new convergence proof of Adomian's technique based on properties of convergent series[6].

Also many outhers used ADM to solve fractional order differential equation [8-11]. Adomian decomposition method has been employed to obtain solutions of a system of fractional differential equations [8] The Adomian decomposition method has been successively used to find the explicit and numerical solutions of the time fractional partial differential equations [9]. Introduces a convergence-control parameter into the standard Adomian decomposition method and establishes a new iterative formula. Analyze the solution of the *n*-term linear fractional-order differential equation with constant coefficients by Adomian decomposition method in [10]. linear differential equations of fractional order has been solved in [11].

Modification on ADM has been done by Many authors and researcher Wazwaz [12] established a new algorithm for calculating Adomian polynomials and introduced the modified ADM to solve meny differential equations. MADM

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technique for dynamic solutions offers an explicit time- marching algorithm that works accurately over such a bigger time step [13]. Based on Newton's method, Abbasbandy [14] presented the modified ADM and applied it to construct the numerical algorithms.

In order to overcome inaccurate terms arising from solving nonlinear differential equations with the higher time derivative, Abassy [15] defined new Adomian polynomials and provided a qualitative improvement over the standard ADM. The Pade´ approximants are effectively used in the analysis to capture the essential behavior of the solution .The combination of the series solution with the Pade´ approximants was successfully implemented in [17–18] and proved to be effective and promising.

II. FRACTIONAL ORDER CALCULUS

The fractional calculus and fractional differential equations have recently become increasingly important topics in the literature of engineering, science and applied mathematics. Application areas include viscoelasticity, electromagnetics, heat conduction, control theory and diffusion [19-23].

Fractional calculus deals with derivatives and integrals of arbitrary order and conceder a generalization of classical calculus. Fractional calculus deals with derivatives and integrals of arbitrary order provides a more powerful tool for modeling the real live phenomena, and this is actually a natural result of the fact that in FC the integer orders are just special cases.

Definition: Let $\alpha \in R^+$. The operator J_a^α defined on $L1[a, b]$ by

$$J_a^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau) d\tau \tag{3}$$

for $a \le t \le b$, is called the Riemann-Liouville fractional integral operator of order α

Definition: Let $\alpha \in R^+$ and $n = \lceil \alpha \rceil$. The operator D_a^α defined as

$$D_a^\alpha f(t) = D^n J_a^{n-\alpha} f(t) \tag{4}$$

$$D_a^\alpha f(t) = \begin{cases} D^n \dfrac{1}{\Gamma(n-\alpha)} \displaystyle\int_a^t (t-\tau)^{n-\alpha-1} f(\tau)\, d\tau & n-1 < \alpha < n \\ \dfrac{d^n}{dt^n} f(t) & \alpha = n \end{cases} \tag{5}$$

for $a \le t \le b$, is called the Riemann-Liouville differential operator of order α.
the Riemann-Liouville differential operator is the left-inverse operator of the Riemann-Liouville fractional integral operator
i.e $D_a^\alpha J_a^\alpha = I$

by convention $D_a^0 f(t) = f(t)$ $i.e$ $D_a^0 = I$

Definition: Let $\alpha \in R^+$ and $n = \lceil \alpha \rceil$. The operator D_{*a}^α defined by

$${}_a^cD_x^\alpha f(t) = D_{*a}^\alpha = \begin{cases} \dfrac{1}{\Gamma(n-\alpha)} \displaystyle\int_a^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)\, d\tau & n-1 < \alpha < n \\ \dfrac{d^n}{dt^n} f(t) & \alpha = n \end{cases} \tag{6}$$

for $a \le t \le b$, is called the Caputo differential operator of order α

The relationship between the caputo derivative and the Riemann-Liouville derivativae is the following.

Theorem 1 let $\alpha > 0$, assume that f is such that both ${}_a^RD_t^\alpha f$ and ${}_a^CD_t^\alpha f$ exist

$${}_a^CD_t^\alpha f(t) = {}_a^RD_t^\alpha f(t) - \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{\Gamma(k+1-\alpha)} (t-a)^{k-a} \tag{7}$$

Proof: note that, Diethelm[23],

$$ {}_a^C D_t^\alpha\, f(t) = {}_a^R D_t^\alpha\, f(t) - \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{\text{k!}}\, {}_a^R D_t^\alpha\, [(.-a)^k](t) $$

$$ {}_a^C D_t^\alpha\, f(t) = {}_a^R D_t^\alpha\, f(t) - \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{\Gamma(k+1-\alpha)}(t-a)^{k-\alpha} $$

Theorem 2 let $\alpha > 0, n-1 < \alpha < n$, then,

$$ {}_a^C D_t^\alpha\, f(t) = {}_a^R D_t^\alpha\, f(t) - \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{\Gamma(k+1-\alpha)}(t-a)^{k-\alpha} $$
$$ = {}_a^R D_t^\alpha\, [f(t) - T_{n-1}[f;a](t)] $$

Here

$$ T_{n-1}[f;a](t) = \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{\text{k!}}(t-a)^k $$

Denotes the Taylor polynomial of degree $n-1$ for the function f, centred at a; in the case $n = 0$ thus we define $T_{n-1}[f;a] = 0$

Proof: from Theorem 1 we have

$$ {}_a^C D_t^\alpha\, f(t) = {}_a^R D_t^\alpha\, f(t) - \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{\Gamma(k+1-\alpha)}(t-a)^{k-\alpha} $$
$$ {}_a^C D_t^\alpha\, f(t) = {}_a^R D_t^\alpha [\, f(t) - T_{n-1}[f;a](t)] $$

Note that if $a = 0$, then

$$ {}_0^C D_t^\alpha\, f(t) = {}_0^R D_t^\alpha [\, f(t) - T_{n-1}[f;a](t)] $$

where

$$ T_{n-1}[f;a](t) = \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{\text{k!}}(t)^k, n-1 < \alpha < n. $$

When, $0 < \alpha < 1, n = 1$, we have

$$ {}_0^C D_t^\alpha\, f(t) = {}_0^R D_t^\alpha [\, f(t) - f(0)] \tag{8} $$

III.ANALYSIS OF THE ADOMIAN DECOMPOSITION METHOD

The method involves splitting an equation into linear and non-linear parts, and then decomposing the solution into an infinite series. This series has to be truncated for practical purposes but by adding more terms it is possible to get arbitrarily close to the exact solution in a specific domain.

To apply the Adomian decomposition method for solving nonlinear ordinary differential equations, we consider the equation

$$ Ly + R(y) + F(y) \;=\; g(t), \tag{9} $$

where the differential operator L may be considered as the highest order derivative in the equation, R is the remainder of the differential operator, $F(y)$ expresses the nonlinear terms, and $g(t)$ is an inhomogeneous term.

If L is a first order operator defined by

$$ L \;= d/\,dt $$

So the inverse operator $L^{-1}$is given by

$$ L^{-1}(.) = \int_0^x (.)dt \tag{10} $$

So that

$$ L^{-1}Ly = \int_0^t \frac{dy}{dt}dt = y(t)|_0^t = y(t) - y(0) \tag{11} $$

if L is a second order differential operator given by

$$ L = \frac{d^2}{dt^2} $$

so that the inverse operator L^{-1} is regarded a two-fold integration operator defined by

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$$L^{-1}(.) = \int_0^t \int_0^t (.)dtdt \tag{12}$$
$$L^{-1}Ly = y(t) - y(0) - ty'(0).$$

In a parallel manner, if L is a third order differential operator, we can easily show That

$$L^{-1}Ly = y(t) - y(0) - ty'(0) - \frac{1}{2!}ty''(0) \tag{13}$$

For higher order operators we can easily define the related inverse operators in a similar way.

Applying L^{-1} to both sides of

$$Ly + R(y) + F(y) = g(t)$$

gives

$$y(t) = \psi_0 + L^{-1}g(t) - L^{-1}Ry - L^{-1}F(y). \tag{14}$$

Where

$$\psi_0 = \begin{cases} y(0), & for\ L = \frac{d}{dt} \\ y(0) + ty'(0), & for\ L = \frac{d^2}{dt^2} \\ y(0) + ty'(0) + \frac{1}{2!}t^2y''(0), & for\ L = \frac{3}{dt^3} \\ y(0) + ty'(0) + \frac{1}{2!}t^2y''(0) + \frac{1}{3!}t^3y'''(0), & for\ L = \frac{d^4}{dt^4} \\ y(0) + ty'(0) + \frac{1}{2!}t^2y''(0) + \frac{1}{3!}t^3y'''(0) + \frac{1}{4!}t^4y^{(4)}(0), & for\ L = \frac{d^5}{dt^5} \end{cases} \tag{15}$$

and so on. The Adomian decomposition method admits the decomposition of y into an infinite series of components

$$y(t) = \sum_{n=0}^{\infty} y_n \tag{16}$$

and the nonlinear term $F(y)$ be equated to an infinite series of polynomials

$$F(y) = \sum_{n=0}^{\infty} A_n \tag{17}$$

where A_n are the Adomian polynomials. Substituting (16) and (17) into (14) gives

$$\sum_{n=0}^{\infty} y_n = \psi_0 - L^{-1}g(t) - L^{-1}R(\sum_{n=0}^{\infty} y_n) - L^{-1}(\sum_{n=0}^{\infty} A_n) \tag{18}$$

The various components y_n of the solution y can be easily determined by using the recursive relation

$$y_0 = \psi_0 - L^{-1}\big(g(t)\big),$$
$$y_{k+1} = -L^{-1}(Ry_k) - L^{-1}(A_k), k \geq 0 \tag{19}$$

Consequently, the first few components can be written as

$$y_0 = \psi_0 - L^{-1}g(t),$$
$$y_1 = -L^{-1}(R\,y_0) - L^{-1}(A_0),$$
$$y_2 = -L^{-1}(R\,y_1) - L^{-1}(A_1),$$
$$y_3 = -L^{-1}(R\,y_2) - L^{-1}(A_2),$$
$$y_4 = -L^{-1}(R\,y_3) - L^{-1}(A_3),$$

Having determined the components yn, n _ 0, the solution y in a series form follows immediately. As stated before, the series may be summed to provide the solution in
a closed form. However, for concrete problems, the n−term partial sum

$$\emptyset_n = \sum_{k=0}^{n-1} y_k$$

may be used to give the approximate solution.
In the following, several examples will be discussed for illustration.

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Apply the decomposition method requires that the fractional differential equation takes form:

$$D_*^\alpha\, y(t) = A(t) + B(t)y + C(t)y^2$$

where the fractional differential operator D_*^α is defined as:

$$D_*^\alpha = \frac{d^\alpha}{dt^\alpha}$$

Applying the operator J^α the inverse of the operator D_*^α to both sides of Eq. (1) and using the initial conditions lead to

$$y(t) = \sum_{j=0}^{m-1} C_j \frac{t^j}{j!} + J^\alpha[A(t) + B(t)y + C(t)y^2] \tag{20}$$

The Adomain's decomposition method suggests the solution y(t) be decomposed by the infinite series of components

$$y(t) = \sum_{n=0}^{\infty} y_n(t) \tag{21}$$

and the nonlinear function in Eq. (20) is decomposed as follows:

$$N(y) = y^2 = \sum_{n=0}^{\infty} A_n \tag{22}$$

where A_n are the so-called the Adomian polynomials.
Substitution the decomposition series (21) and (22) into both sides of (20) gives

$$\sum_{n=0}^{\infty} y_n(t) = \sum_{j=0}^{m-1} C_j \frac{t^j}{j!} + J^\alpha\left[A(t) + B(t)\sum_{n=0}^{\infty} y_n(t) + C(t)\sum_{n=0}^{\infty} A_n(t)\right] \tag{23}$$

From this equation, the iterates are determined by the following recursive way:

$$\begin{aligned} y_0 &= \sum_{j=0}^{m-1} c_j \frac{t^j}{j!} + J^\alpha(A(t)) \\ y_{n+1} &= J^\alpha(B(t)y_n + C(t)A_n(t)) \qquad n \geq 0 \end{aligned} \tag{24}$$

The values of the natural number m can be 1 and 2 corresponding to $0 < a < 1$ and $1 < a < 2$, respectively.
The K-term approximate solution is then defined as

$$\emptyset_K = \sum_{m=0}^{K-1} y_m(t), \tag{25}$$

and the exact solution is

$$y(t) = \lim_{K\to\infty} \emptyset_K(t) \tag{26}$$

However, in many cases the exact solution in a closed form may be obtained. Moreover, the decomposition series solutions are generally converge very rapidly. The convergence of the decomposition series has investigated by several authors.

IV. COMBINATION BETWEEN ADM AND RESTRICTIVE PADÉ

Henri Eugène Padé (December 17, 1863 – July 9, 1953) was a French mathematician, who is now remembered mainly for his development of approximation techniques for functions using rational functions. The Padé approximants are a particular type of rational fraction approximation to the value of the function [24-32].

The Padé approximant often given as:

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$$H(s) = \frac{A(s)}{B(s\,)} \tag{27}$$

The Padé approximation can be written in the form

$$PA[M/N]_{f(x)}\,(x) = \frac{\sum_{i=0}^{M} a_i x^i}{1+\sum_{i=1}^{N} b_i x^i} \quad \textit{where M and N are positive integers} \tag{28}$$

There are $M + 1$ independent numerator coefficients and N denominator coefficients making $M + N + 1$ unknown coefficients. The idea is to match the Taylor series expansion as far as possible

The $M + N + 1$ unknown suggests that normally the $PA\ [M/N]$ ought to fit the power series $f(x) = \sum_{i=0}^{\infty} c_i x^i$.

Ismail et al. [24-30] applied Restrictive Padé approximation to solve many differential equations. Function approximation that meet Taylor approximation was done by Ismail et al. in [31] . In paper [32] Ismail et al. introduce VIM- Restrictive Padé approximation.

The restrictive Padé approximation is a rational function in the form:

$$RPA[M+\alpha/N]_{f(x)}\,(x) = \frac{\sum_{i=0}^{M} a_i x^i + \sum_{i=1}^{\alpha} \varepsilon_i x^{M+i}}{1+\sum_{i=1}^{N} b_i x^i} \tag{29}$$

where the positive integer α does not exceed the degree of the numerator, $\alpha = 0(1)n$ Such that

$$f(x) = RPA[M+\alpha/N]_{f(x)}\,(x) + O(X^{M+N+1}) \tag{30}$$

There are two main steps first we find the traction series given by ADM finally convert this series to Restrictive Padé approximation

V. NUMERICAL EXAMPLE

Consider the following fractional riccati equation:

$$\frac{d^\alpha y}{dt^\alpha} = -y^2(t) + 1, \qquad 0 < \alpha \leq 1 \tag{31}$$

Subject to the initial condition

$$y(0) = 0$$

The exact solution, when $\alpha = 1$, is

$$y(t) = \frac{e^{2t} - 1}{e^{2t} + 1} \tag{32}$$

The Adomain solution

$$y_0 = y(0) + J^\alpha(1) = \frac{1}{\Gamma(\alpha+1)} t^\alpha$$
$$y_{n+1} = -J^\alpha(A_n) \qquad n \geq 0$$

where A_n are Adomian polynomials for the nonlinear term $F(y) = y^2$.

Using the above recursive relationship and Mathematica, the first few terms of the decomposition series are given by

$$y_0 = \frac{1}{\Gamma(\alpha+1)} t^\alpha$$
$$y_1 = -J^\alpha(y_0^2) = -\frac{\Gamma(1+2\alpha)}{\alpha^2\Gamma(1+3\alpha)} t^{3\alpha}$$
$$y_2 = -J^\alpha(2y_0y_1) = \frac{16\Gamma(2\alpha)\Gamma(4\alpha)}{\alpha\Gamma(1+3\alpha)\Gamma(1+5\alpha)} t^{5\alpha}$$
$$y_3 = -J^\alpha(2y_0y_2 + y_1^2) = -\frac{\left(32\alpha^2\Gamma(2\alpha)\Gamma(4\alpha)\Gamma(1+3\alpha) + \Gamma(1+2\alpha)^2\Gamma(1+5\alpha)\Gamma(1+5\alpha)\right)}{\alpha^4\Gamma(1+3\alpha)^2\Gamma(1+5\alpha)\Gamma(1+7\alpha)} t^{7\alpha}$$

$$y(t) = y_0 + y_1 + y_2 + \cdots$$

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First order case: setting $\alpha = 1$

$$y_2(t) = t - 0.333333\, t^3 + 0.133333\, t^5 \tag{33}$$

$$PA[2/2] = \frac{0. + 1.t}{1. + 0.333333t^2} \tag{34}$$

Taking the series solution on the form:

$$f(\mathrm{t}) = t - 0.333333t^3 + 0.133333t^5$$

the coffic. that equal Taylor series is:

$$c_0 = 0$$
$$c_1 = 1$$
$$c_2 = 0$$
$$c_3 = -0.333333$$

$$\begin{aligned} a_0 &= c_0 \\ a_1 &= c_1 + \frac{c_0c_1c_2 + c_0c_0c_2 - c_0c_1\varepsilon_1}{c_0c_2 - c_1c_1} \\ a_2 &= \varepsilon_1 \\ b_1 &= \frac{c_1c_2 - c_0c_3 - c_1\varepsilon_1}{c_0c_2 - c_1c_1} \\ b_2 &= \frac{c_1c_3 - c_2c_2 - c_2\varepsilon_1}{c_0c_2 - c_1c_1} \end{aligned} \tag{35}$$

The system of linear equations (35) has only one unknown that called the restrictive term which forced the require series to fit the approximated fractional function (RPA), this unknown valid by finish the equation:

$$f(t) - RPA[m + \alpha/n]_{f(t)}\,(t) = 0 \tag{36}$$

For t=0.7

$$f[0.7] - \left(\frac{\mathrm{a0} + \mathrm{a1}(0.7) - +\epsilon(0.7)^2}{1 + \mathrm{b1}(0.7) + \mathrm{b2}(0.7)^2}\right) = 0$$
$$\epsilon = 0.11492495673085872$$

$$RPA\left[\frac{1+1}{2}\right] = \frac{1.x + 0.11492495673085872x^2}{1.0 + 0.11492495673085872x + 0.333333x^2} \tag{37}$$

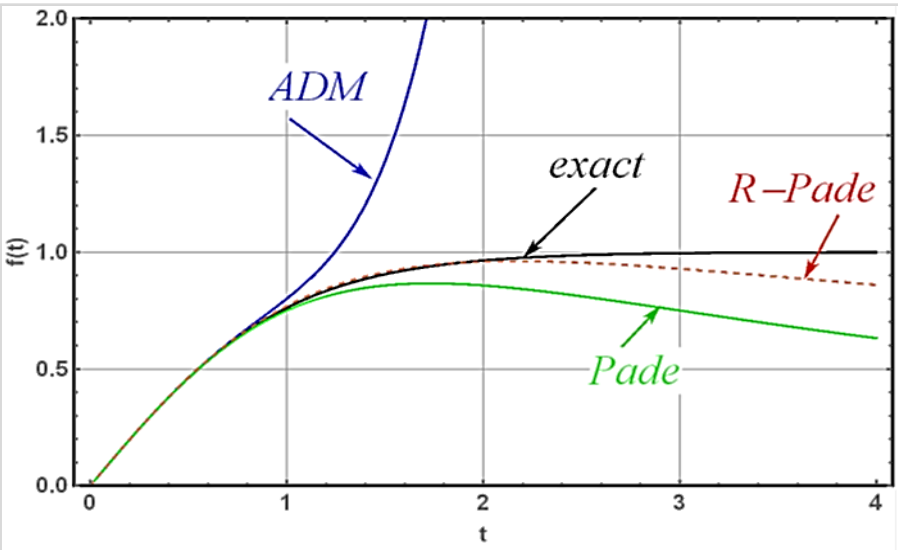

Fig. 1 Comparison between ADM, Padé -ADM and Restrictive Padé -ADM for $y_2(t)\ at\ \alpha = 1$

Fractional order case : in this case we will examine the fractional ricatti equation setting $\alpha = \frac{1}{2}$ gives

$$y(t) = 2\sqrt{t} - 3.00901\ t^{\frac{3}{2}} + 7.24332\, t^{\frac{5}{2}} \tag{38}$$

For integer power to approximate Padé and restrictive Padé Let $t^{\frac{1}{2}} = x$, then

$$y(x) = 2x - 3.00901\ x^3 + 7.24332\, x^5 \tag{39}$$

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$$PA[\frac{2}{2}]_{f(x)} = \frac{2.x}{1. + 1.504505x^2}$$

Reverse to get $PA[2/2]_{f(t)}$ by $x = t^{\frac{1}{2}}$

$$PA[2/2]_{f(t)} = \frac{2.\sqrt{t}}{1. + 1.504505\ \mathrm{t}} \quad (40)$$

To find restrictive pade approximation from equation (39).

$$c_0 = 0, \quad c_1 = 2, \quad c_2 = 0\,, \quad c_3 = -3.00901$$

$$\begin{aligned} a_0 &= c_0 \\ a_1 &= c_1 + \frac{c_0c_1c_2 + c_0c_0c_2 - c_0c_1\varepsilon_1}{c_0c_2 - c_1c_1} \\ a_2 &= \varepsilon_1 \\ b_1 &= \frac{c_1c_2 - c_0c_3 - c_1\varepsilon_1}{c_0c_2 - c_1c_1} \\ b_2 &= \frac{c_1c_3 - c_2c_2 - c_2\varepsilon_1}{c_0c_2 - c_1c_1} \end{aligned} \quad (41)$$

The system of linear equations (41) has only one unknown that called the restrictive term which forced the require series to fit the approximated fractional function (RPA), this unknown valid by finish the equation:

$$f(t) - RPA[m + \alpha/n]_{f(t)}(t) = 0$$

For t=0.5

$$f[0.5] - (\frac{\mathrm{a0} + \mathrm{a1}(0.5) + \epsilon(0.5)^2}{1 + \mathrm{b1}(0.5) + \mathrm{b2}(0.5)^2}) = 0$$

$$\epsilon = 4.54076365746382$$

$$RPA[2/2]_{f(x)} = \frac{2.x + 4.54076365746382x^2}{1.0 + 2.27038182873191x + 1.504505x^2}$$

Reverse to get $RPA[2/2]_{f(t)}$ by $x = t^{\frac{1}{2}}$

$$RPA[2/2]_{f(t)} = \frac{2.t^{\frac{1}{2}} + 4.54076365746382t}{1.0 + 2.27038182873191t^{\frac{1}{2}} + 1.504505t} \quad (42)$$

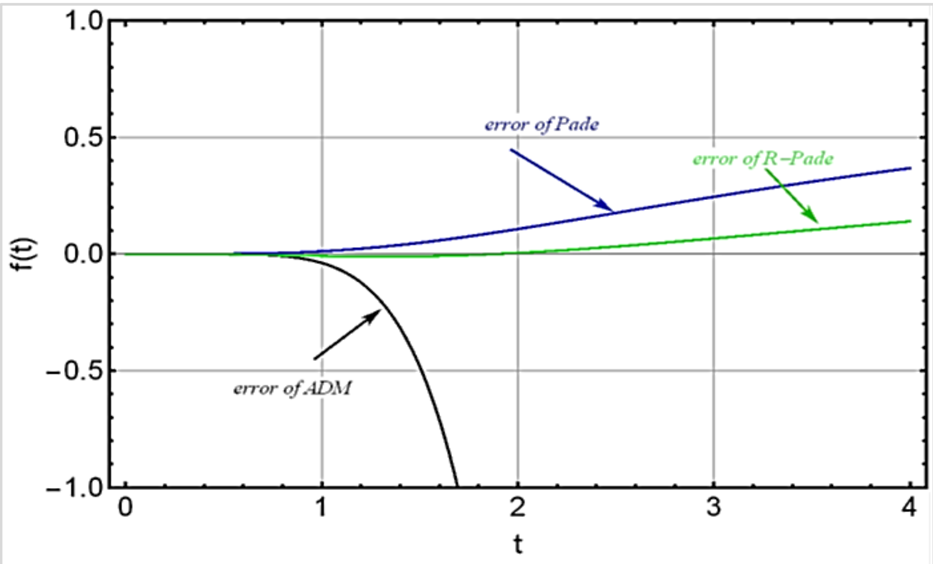

Fig.2 Error between solution ,ADM, Padé -ADM and Restrictive Padé -ADM for $y_2(t)\ at\ \alpha = 1$

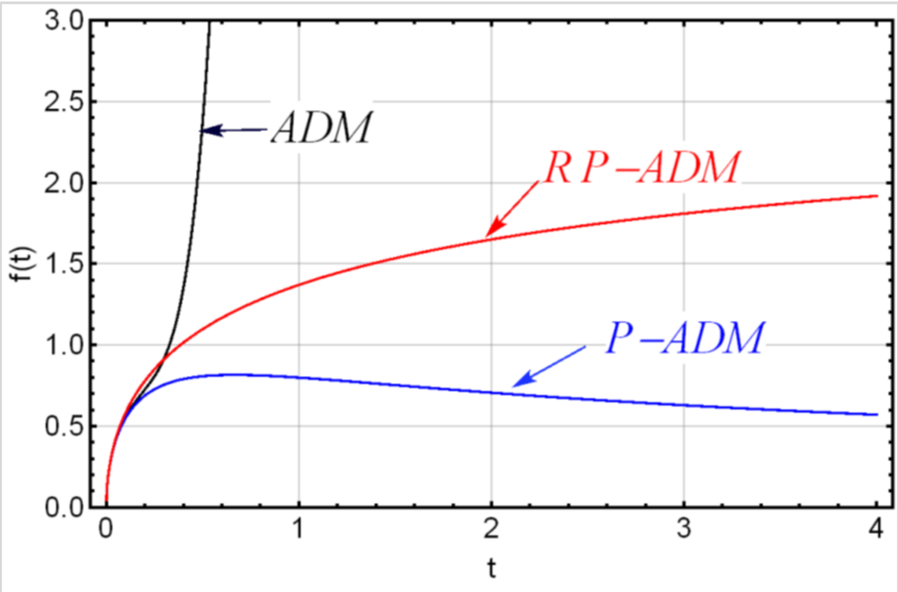

Fig. 3 Comparison between ADM, Padé -ADM and Restrictive Padé -ADM for $y_2(t)\ at\ \alpha = 0.5$

VI. RESULTS AND CONCLUSION

In this paper, the Modification on ADM is employed to solve Fractional Riccati Differential Equation. The solution of differential equation for both fractional and classical has been solved using ADM and show acceptable result, effective and very simple. Here we used ADM to have the series solution then converte this solution to classical pade approximation which better than the smi-anlaytical solution finally we apply the restrictive Padé approximation on the series obtained by ADM and called this solution Adomian decomposition method- restrictive Padé (ADM-RP) ,this solution give more accurate, less calculation and less error.

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